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On the $m_\perp$-dependence of Bose-Einstein correlation radii

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Abstract

Within a theoretical study using both HYLANDER and RQMD, we revisit Bose-Einstein correlation measurements of 200AGeV S+S and 160AGeV Pb+Pb. Transverse flow does not show up in the Bose-Einstein measurements. The decrease of the effective transverse radius parameters with increasing transverse average momentum of the particle pair is largely due to the effect of resonance decays.

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In the ongoing search for the quark-gluon plasma (QGP), Bose-Einstein correlations (BEC) of identical bosons have become an issue of great current interest. BEC serve as a tool in the determination of radii (spatial extensions) and lifetimes (temporal extensions) of hadron emission zones generated in relativistic nucleus-nucleus collisions [1]. However, the inverse widths of BEC functions do not have a simple interpretation in terms of those radii and lifetimes. The inverse widths or "correlation radii" have to be considered as combinations of space-time moments from space-time densities (*cf.*, e.g., ref. [2] and references therein). The phase-space distribution, or the source function $g(x,p)$, does not only depend on the space-time point x and the four-momentum p of the emitted particles, but also on the temporal evolution of the hadron source (fireball) and, more importantly, on the correlations among x and p [3]. Furthermore, it is known that BEC are sensitive to the effects of the decay of unstable particles (resonances). Experimentally obtained BEC functions are always averaged over certain phase-space regions and are therefore in general sensitive to the specific kinematical regions under consideration. Because of the complications involved in BEC, it is necessary to account for all known effects before we can draw conclusions about new physics which might emerge in relativistic nucleus-nucleus collisions.

Recent theoretical studies [4]-[7] claim that the inverse widths of BEC functions in relativistic heavy-ion collisions at CERN/SPS energies near 200 GeV per nucleon show a $m_\perp$-scaling behavior, namely that the size parameters are inversely proportional to the square-root of the transverse mass, $\sqrt{m_\perp}$. This idea was proposed earlier by Sinyukov et al. [8] in 1987, in which the authors suggested that the longitudinal size parameter, $R_\parallel$, should follow the $m_\perp$-scaling:

$$R_\parallel \propto \sqrt{\frac{T_f}{m_\perp}\frac{1}{\cosh(y_K)}} \;, \tag{1}$$

where T_f and y_K are the particle freeze-out temperature and rapidity of the particle pair, respectively. The pair transverse mass is defined as $m_\perp = \sqrt{m^2 + \vec{K}_\perp^2}$ with the average transverse momentum $\vec{K}_\perp = (\vec{p}_{1\perp} + \vec{p}_{2\perp})/2$ of the particle pair. This relationship has been indeed observed in the longitudinal size parameter $R_\parallel$ from the sulphur-induced high energy collisions [9] and the effect is attributed to the collective expansion. Neglecting the resonance decays, the authors in refs. [4]-[7] suggested that even in the transverse directions, size parameters R_{out} and R_{side} also follow the same $m_\perp$-scaling law. On the other hand, it has been shown that the effect of the resonance decay strongly affect the inverse widths of the BEC [10]-[12].

It is the purpose of this short note to study the $m_\perp$-scaling behaviour by using two realistic source models: HYLANDER [13] and RQMD [14]. This study includes effects of transverse expansion and decay of resonances on BEC of identical particles. We shall first compare HYLANDER results with recent measurements of S+S and Pb+Pb collisions. Then we will focus on the $m_\perp$-scaling issues by inspecting the $m_\perp$-dependence of the size parameters calculated from both models.

The HYLANDER model belongs to the class of models applying (3+1)-dimensional relativistic one-fluid-dynamics (for other hydrodynamical models *cf.* [15] and refs. therein). HYLANDER provides fully three-dimensional numerical solutions of the hydrodynamical relativistic Euler-equations [16]. The model has been successfully applied to several different heavy-ion collisions at SPS energies [17]. Once the initial conditions and the equation of state are specified, one obtains an unambiguous solution from the hydrodynamical equations. The choice of a freeze-out condition such as, *e.g.*, that the expanding fluid reaches a fixed freeze-out temperature, T_f, determines the final space-time geometry of the hydrodynamically expanding fireball. In particular, for 200AGeV S+S and 160AGeV Pb+Pb central collisions, the calculation of single inclusive spectra of negative hadrons and protons and BEC of negative pions and kaons was performed using the formalism outlined in refs. [10, 11, 18]. Resonance decays were included in these calculations. The results for BEC of negative pions and kaons have been published in refs. [10]-[12],[15].

In order to extract effective hadron source radii, the results of the BEC calculations have been fitted to the Gaussian form[1] which has been widely used by experimentalists for the presentation of their BEC data (for the choice of the variables, *cf.* ref. [20]):

$$C_2(\vec{p}_1,\vec{p}_2) \;=\; 1 \;+\; \lambda \,\exp\left[-\frac{1}{2}\left(q_\parallel^2 R_\parallel^2(\vec{K}) \;+\; q_{side}^2 R_{side}^2(\vec{K}) \;+\; q_{out}^2 R_{out}^2(\vec{K})\right)\right] \;. \qquad (2)$$

In eq. (2) $\vec{p}_1$, $\vec{p}_2$ are the momenta of the two emitted identical particles, whereas $\vec{K}$ denotes the average momentum of the particle pair. The q_i ($i = \|, side, out$) are components of the momentum difference vector $\vec{q}$ of the particle pair. It should be emphasized that in the present model $\lambda \equiv \lambda(\vec{K})$ does *not* represent the effect of coherence, but the momentum-dependent effective reduction of the intercept due to the contributions from the decay of long-lived resonances (such as η, η' ...

[1] If one performs a fit to a BEC function in more than one dimension, eq. (2) does not represent the most general expression, because of the existence of an "out-longitudinal" cross term [19].

etc.) [10]-[12],[15]. A characteristic property of particle production from an expanding source is a correlation between the space-time point where a particle is emitted and its energy-momentum [3]. As a consequence, the inverse widths $R_i(\vec{K})$ ($i = \|, side, out$) extracted from Bose-Einstein correlation functions show a characteristic dependence of the average momentum of the pair, $\vec{K}$, and are therefore dependent on detector acceptances.

Fig. 1 (solid lines) shows the calculations for the effective radii $R_{\|}$, R_{side} and R_{out} as functions of the transverse momentum $K_\perp$ of the pion pair compared to the corresponding NA35 and preliminary NA49 data [9, 21], respectively. In order to make a comparison of the calculated effective radii with the experimentally obtained ones, detector acceptances have been accounted for. In the case of 200AGeV S+S the effective radii have been calculated as functions of $K_\perp$ at the rapidity of the pair $y_K = 4.0 - y_{cm} \approx 1.0$. In the case of 160AGeV Pb+Pb the effective radii were calculated as functions of $K_\perp$ at $y_K = 4.5 - y_{cm} \approx 1.6$. The effective longitudinal radii $R_{\|}$ are evaluated in the longitudinal comoving system (LCMS), with $\gamma_{\|} = \cosh(y_K)$. All of these calculations, which in the case of S+S had been true predictions, agree surprisingly well with the data.

The effective radii of only the directly emitted (thermal) negative pions, π^-, (dashed lines) are also plotted in Fig. 1. We can see that the effective radii of the directly emitted π^- are smaller compared to those of all π^-, *i.e.*, including the pions originating from resonance decays, and follow a *different* $K_\perp$-dependence. The effective radii for all pions are larger compared to the effective radii of directly emitted pions, because the unstable particles (resonances) have a finite lifetime. Within their lifetime the resonances can propagate to a distant location from their points of origin (the fireball). While decaying the resonances generate a cloud of pion emission locations around the actual fireball and therefore increase the effective size of the system.

Fig. 1 shows that the $K_\perp$-dependence of the effective transverse radii of thermal pions is very weak while the effective transverse radii of all pions significantly decrease as a function of $K_\perp$. In refs. [12, 15] it was explained that the hydrodynamical solutions express transverse expansion. The two systems each have a maximum value for the transverse velocities: for 200AGeV S+S we have a maximum transverse velocity $u_\perp^{max}(S) = 0.43$ whereas for 160AGeV Pb+Pb we have $u_\perp^{max}(Pb) = 0.61$. The maximal values for the transverse velocities correspond to fluid cells which have space-time rapidities $\eta \approx 0.5 \cdot \ln(t + z/t - z)$ equal to zero (t and z are the time and the

longitudinal spacial coordinate, respectively). On the contrary, fluid cells with large space-time rapidity have very small corresponding values for the transverse velocities at freeze-out (*cf.* Fig. 6 in ref. [15]). We have checked that in the phase-space regions, where the BEC data of the NA35 and NA49 Collaborations have been taken, the contributions to the BEC functions come mainly from fluid cells with almost vanishing transverse velocity at freeze-out.

Transverse flow has two major effects on effective BEC radii. First there is an apparent reduction of the effective longitudinal radii, because the fluid also expands transversely. Secondly, the transverse effective radii show a $K_\perp$-dependence. For the particular hydrodynamical solutions under consideration transverse flow results in a decrease of the transverse effective radii due to the effects of relatively fast transverse inwardly moving rarefaction waves. In agreement with ref. [18], the effective transverse radii of directly emitted negative pions plotted in Fig.1 show almost no $K_\perp$-dependence. But the effective radii of all π^- have a $K_\perp$-dependence which would indicate a present transverse flow in the data. Since there is no transverse flow in the considered phase-space region as we have argued above, the decrease of the effective radii for all π^- must have a different reason. In fact, for smaller transverse momenta we have relatively larger contributions from resonance decay to the BEC than at higher transverse momenta $K_\perp$. Since the relative weight of pions being produced directly compared to those originating from resonance decays enters in the composition of BEC functions, the decrease of the effective radii of all pions with increasing $K_\perp$ can be understood in terms of the chemical composition at freeze-out as a function of particle momenta. Resonance decays produce a pionic cloud that surrounds the fireball. This pion cloud has in general larger spatial extensions (and a longer lifetime) than the fireball which emits the direct pions. Thus relatively larger resonance contributions result in relatively larger correlation radii and therefore, the decay of resonances is responsible for the decrease of the transverse radii with increasing $K_\perp$.

Let us now discuss $m_\perp$-scaling according to refs. [4]-[7]. If $m_\perp$-scaling would be present, we would get

$$\gamma_\parallel^2 R_\parallel^2(\vec{K}) \;\simeq\; R_{side}^2(\vec{K}) \;\simeq\; R_{out}^2(\vec{K}) \;\propto\; \frac{T_f}{m_\perp}. \tag{3}$$

The HYLANDER BEC calculations for the transverse radii R_{out} and R_{side} show an even stronger decrease with $K_\perp$ at midrapidity, i.e., $y_K = 0.0$ (*cf.* Fig. 2 in ref. [10] and Fig. 3a in ref. [12]).

In order to get a maximum effect due to a possible $m_\perp$-scaling we show in Fig. 2 effective radii calculated at $y_K = 0.0$ and multiplied with $\sqrt{m_\perp/T_f}$ as a function of the pair transverse mass $m_\perp$. A present $m_\perp$-scaling would result in a constant behaviour of the plotted quantities, *i.e.*, there would be no change with changing $m_\perp$. Obviously, the effective radii multiplied with the factor $\sqrt{m_\perp/T_f}$ do not show an independence of $m_\perp$. Consequently, the effective radii $R_\parallel$, R_{side} and R_{out} (and those of only directly emitted π^-) are not consistent with a simple $m_\perp$-scaling behaviour in this hydrodynamical model.

Up to now, we have based all of our arguments on the hydrodynamical model HYLANDER and simple $m_\perp$-scaling is found neither in the longitudinal direction, $R_\parallel$, nor in the transverse directions, R_{out} and R_{side}. In addition, we found that resonance decays increase the size parameters at low transverse momenta dramatically.

To further understand the physics of the transverse momentum dependence of the size parameters, we made an additional test with the widely used transport model RQMD (v2.2) [22]. Using the freeze-out phase-space distribution, $g(x,p)$, we calculated the averaged pion size parameters $R_\parallel$, R_{out}, and R_{side} as a function of the pair transverse mass $m_\perp$ for 160AGeV Pb+Pb central collisions, multiplied with $\sqrt{m_\perp/T_f}$ and again at midrapidity, $y_K = 0.0$. The results are shown in Fig. 3 where the solid lines represent all pions and dotted lines are for rescattered pions. Rescattered pions do not originate from resonance decay at the moment of their last interaction. A temperature parameter of $T_f = 140 MeV$ is used here [22]. The rescattered pions are similar to the thermal pions from the hydrodynamical model calculations (see dashed lines in Figs. 1,2), because they are emitted earlier.

It is well known by now that, due to rescatterings in the heavy-ion collisions, collective flow manifests itself in the cascade type of calculations [23, 24]. In the Pb+Pb central collisions, the averaged transverse flow velocity at midrapidity is about 40% of the speed of light. If the $m_\perp$-scaling [4] is correct, one would expect a flat distribution of the scaled size parameters $\sqrt{m_\perp/T_f}R_i$ ($i = \parallel, out, side$) with respect the pair transverse mass, $m_\perp$. However, Fig. 3 does not show such a strong flow effect as proposed in ref. [4]. It is worth noting, that in a recent publication [25] on S+Pb central collisions one indeed observed a $m_\perp$-scaling. The corresponding RQMD predictions which accounted for the exact experimental acceptance were consistent with the data. However,

the experimental acceptance has a rapidity span larger than one unit and, at each transverse momentum, the corresponding rapidity is different. Such an experimental effect may explain the different $m_\perp$-behaviour between the experimental results of NA35 [9] and NA44 [25] and also the difference between our results in Fig. 3 and NA44.

To summarize, we have revisited Bose-Einstein correlation measurements for 200AGeV S+S and 160AGeV Pb+Pb. For the theoretical analysis we have used a hydrodynamical model (HYLANDER) and a cascade model (RQMD). Within the experimental acceptance, the hydrodynamical model results are consistent with the measured size parameters for both 200 AGeV S+S and 160 AGeV Pb+Pb central collisions. However, at midrapidity, no $m_\perp$-scaling is seen in the results from both model calculations. Resonance decays seem to play an important role in the Bose-Einstein correlation radii. Our conclusion is also consistent with recent publications [26, 27].

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Figure Captions

Fig. 1 Effective radii extracted from Bose-Einstein correlation functions of identical pions as a function of the transverse average momentum $K_\perp$ of the pion pair at rapidity $y_K = 4.0 - y_{cm} \approx 1.0$ for 200AGeV S+S and at rapidity $y_K = 4.5 - y_{cm} \approx 1.6$ for 160AGeV Pb+Pb, respectively. Solid lines correspond to all π^-, *i.e.*, including those from resonance decay, whereas the dashed lines correspond to directly emitted (thermal) π^-. The theoretical results are compared to NA35 data [9, 21] and preliminary NA49 data [9].

Fig. 2 Effective radii extracted from Bose-Einstein correlation functions of identical pions multiplied with $\sqrt{m_\perp/T_f}$ as a function of the transverse mass $m_\perp$ of the pion pair at rapidity $y_K = 0.0$ for 200AGeV S+S and for 160AGeV Pb+Pb, respectively. Solid lines correspond to all π^-, *i.e.*, including those from resonance decay, whereas the dashed lines correspond to directly emitted (thermal) π^- only.

Fig. 3 Scaled radii from the transport model RQMD (v2.2). A freeze-out temperature of 140 MeV is used in the figure. Similar to Fig. 2, one does not find the $m_\perp$-scaling law in these radius parameters.

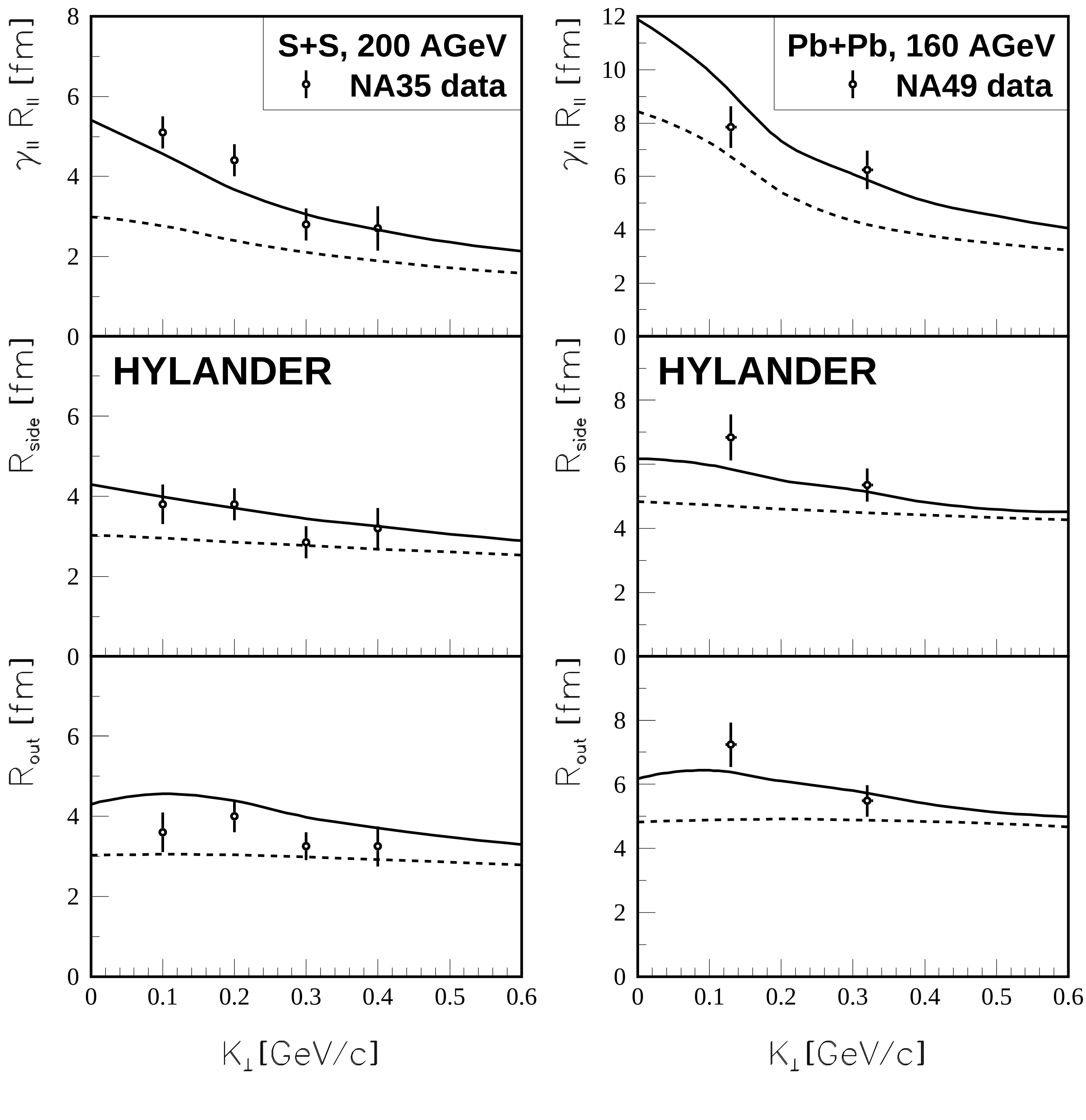
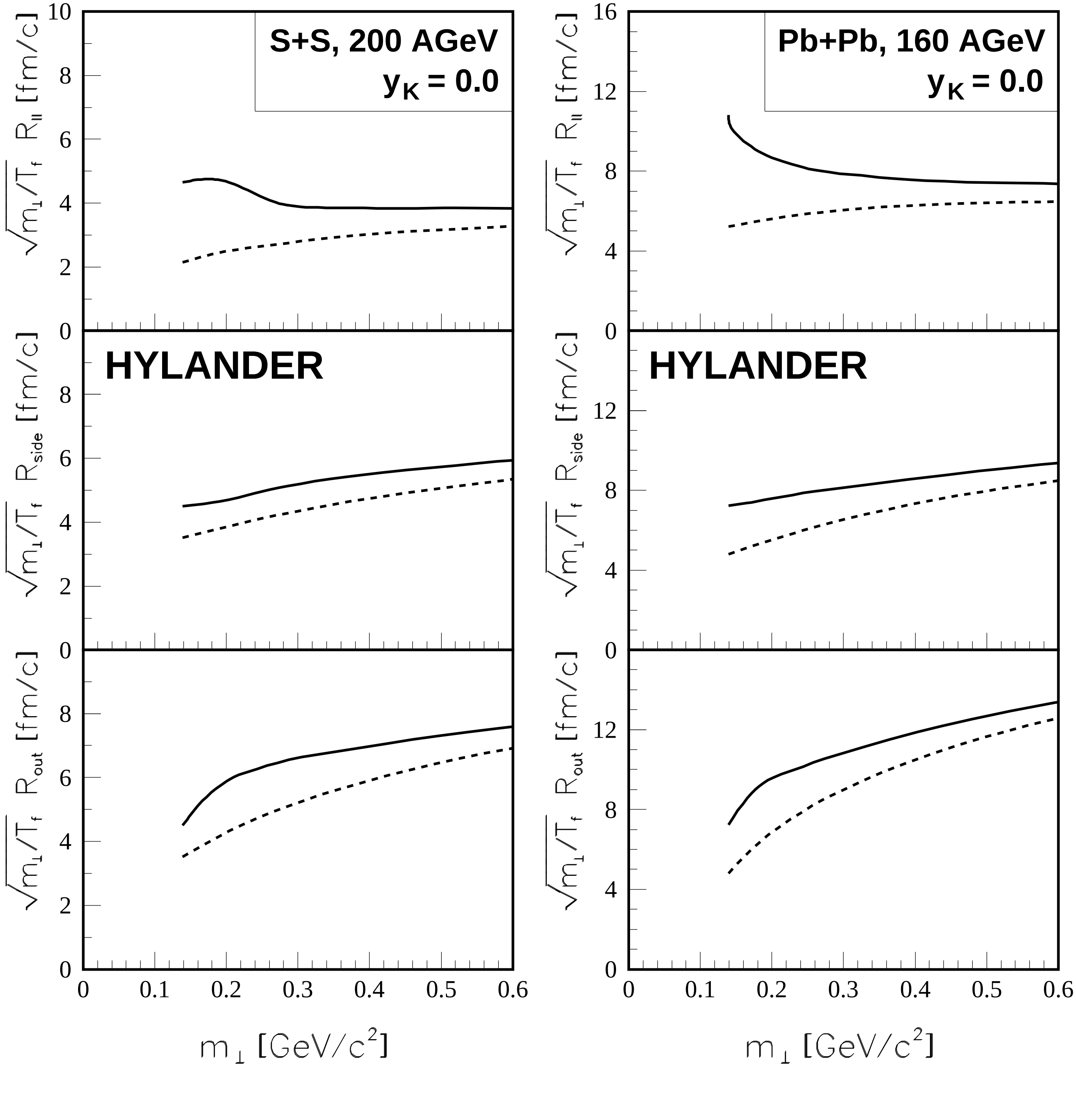

S+S, 200 AGeV
NA35 data
Pb+Pb, 160 AGeV
NA49 data
HYLANDER
HYLANDER
$\gamma_{\parallel} R_{\parallel}$ [fm]
R_{side} [fm]
R_{out} [fm]
$K_{\perp}$[GeV/c]
$K_{\perp}$[GeV/c]

Figure 1



S+S, 200 AGeV
$y_K = 0.0$
Pb+Pb, 160 AGeV
$y_K = 0.0$
HYLANDER
HYLANDER
$\sqrt{m_\perp/T_f}\ R_\parallel$ [fm/c]
$\sqrt{m_\perp/T_f}\ R_{side}$ [fm/c]
$\sqrt{m_\perp/T_f}\ R_{out}$ [fm/c]
$m_\perp$ [GeV/c^2]

Figure 2

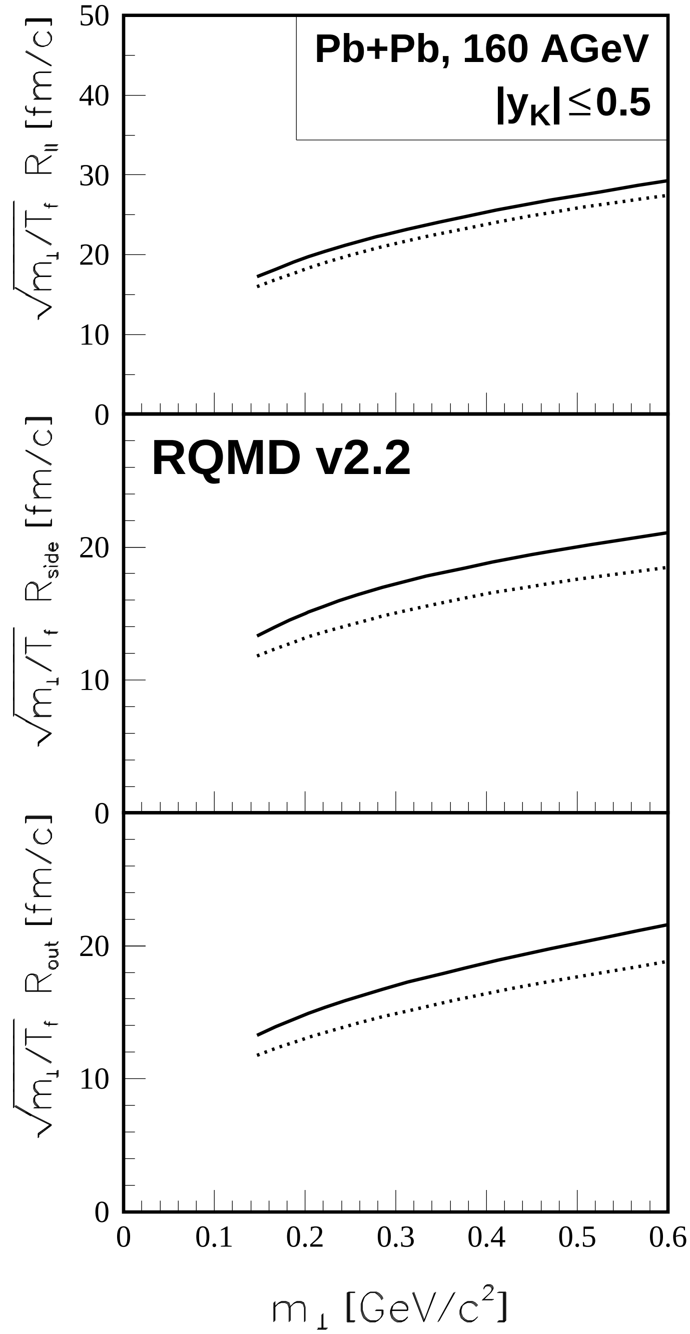

Pb+Pb, 160 AGeV
|y_K|≤0.5
RQMD v2.2
√m⊥/T_f R_∥ [fm/c]
√m⊥/T_f R_side [fm/c]
√m⊥/T_f R_out [fm/c]
m⊥ [GeV/c²]
0
10
20
30
40
50
0.1
0.2
0.3
0.4
0.5
0.6

Figure 3